International Journal of Theoretical Physics, Vol. 41, No. 11, November 2002 (© 2002)

Effective Gravity in Randall–Sundrum Infinite Brane World

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Received August 19, 2002

The gravity induced on the brane in the Randall–Sundrum (RS) infinite brane world is briefly reviewed. We also discuss the possibility of the absence of black hole configuration in this model based on the argument of the AdS/CFT correspondence.

KEY WORDS: brane-world; black hole; AdS/CFT correspondence.

1. INTRODUCTION

Current candidates for the fundamental theory of particle physics, such as string theory or M-theory, are all defined as a theory in higher dimension. To obtain an appropriate 4D effective theory starting with such a theory, a certain dimensional reduction is necessary. One well-known scheme of dimensional reduction is the Kaluza–Klein compactification. In this scheme, the size of the extra dimension is supposed to be very small so as not to excite the modes which have momentum in the direction of the extra dimension. This scheme seems to work well as a mechanism to shield the effect of extra dimensions. This Kaluza–Klein scheme, however, is not a unique possible scheme for dimensional reduction. Recently, the brane-world scenario has been attracting a lot of attention as an alternative possibility (Antoniadis *et al.*, 1998; Arkani-Hamed *et al.*, 1998; Horana and Witten, 1996a,b; Randall and Sundrum, 1999a,b). The essential feature of the brane world distinct from the ordinary Kaluza–Klein compactification is that the matter fields of the standard model are supposed to be localized on the brane, while the graviton can propagate in a higher dimensional space–time which we call "bulk." Owing to the assumption that the ordinary matter fields are localized on the brane, the brane-world models can be consistent with the particle physics experiments even if the length scale of the extra dimension is not extremely small. Then, the gravity is possibly altered at a rather macroscopic length scale, while the observational constraint about the deviation from the 4D general relativity obtained so far is

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not severely below sub-mm scale. Hence, a characteristic length scale can be as large as sub-mm scale in the brane world scenario. Therefore it may open up the possibility of observing the evidence of the existence of an extra dimension.

In the course of studies on brane-world, a new scenario was proposed by Randall and Sundrum (RS, 1999a,b). One of the novel ideas of their new proposal is that the gravity can be effectively localized as a result of the warped compactification, even though the extension of the extra dimension is infinite. Although the recovery of the 4D general relativity in this model is not so automatic, so far any results which are significantly distinguishable from the standard ones have not been reported. In this paper, we review the current status of the studies on the gravity in this model.

This paper is organized as follows. In Section 2 we explain the setup of the RS model with infinite extra dimension. In Section 3 we review the geometrical approach to the gravity in this model, finding the limitation of the approach in which we do not solve the 5D equations of motion. In Sections 4 and 5 we summarize the results for linear perturbations and for nonlinear perturbations of this model, respectively. In Section 6 we discuss the possibility that there is no static black hole solution in this model, applying the argument of the AdS/CFT correspondence. In Section 7 we give a brief summary.

2. WARPED EXTRA DIMENSION

In this section, we explain the model proposed by Randall and Sundrum (1999b). In this model, 5D Einstein gravity with negative cosmological constant Λ is assumed. The ordinary matter fields are confined on a 4D object called "brane." This brane has positive tension σ, and the space–time has reflection symmetry (Z_2-symmetry) at the position of this brane $y = y_b$. Here y is the Gaussian normal coordinate in the direction perpendicular to the brane. The 5D Einstein equations are

$$ {}^{(5)}G_{ab} = -\Lambda g_{ab} + 8\pi G_5 S_{ab} \delta(y - y_b), \tag{1} $$

with

$$ S_{ab} = -\sigma \gamma_{ab} + T_{ab}, \tag{2} $$

where T_{ab} is the energy–momentum tensor of the matter field localized on the brane, γ_{ab} is the 4D metric induced on the brane, and G_5 is the 5D Newton's constant.

One solution of (1) is 5D anti-de Sitter (AdS) space

$$ ds^2 = dy^2 + e^{-2|y|/\ell}(-dt^2 + d\mathrm{x}^2), \tag{3} $$

with a single positive tension brane located at $y = 0$. Here, ℓ is the curvature radius of 5D AdS space. The 5D cosmological constant and the brane tension are set to $\Lambda = -6/\ell^2$ and $\sigma = 3/(4\pi G_5 \ell)$. It is convenient to introduce conformal coordinates defined by $z = \mathrm{sign}(y)\ell(e^{|y|/\ell} - 1)$. In these coordinates the metric is expressed as

$$ds^2 = \frac{\ell}{(|z| + \ell)^2}\left(dz^2 + \eta_{\mu\nu}\, dx^\mu\, dx^\nu\right), \tag{4}$$

where $\eta_{\mu\nu}$ is the 4D Minkowski metric. The outstanding feature of this model is that the 4D general relativity is seemingly reproduced as an effective theory on the brane, although the extention in the direction of the extra dimension is infinite.

3. GEOMETRICAL APPROACH

A quick way to see why the 4D general relativity is expected to be realized on the brane will be the geometrical approach introduced by Shiromizu *et al.* (2000). We use the 4+1 decomposition of the 5D Einstein tensor. The components parallel to the brane are decomposed by the Gauss equation as

$$\begin{aligned} {}^{(4)}G_{\mu\nu} = {}^{(5)}G_{\mu\nu} + {}^{(5)}R_{yy}\gamma_{\mu\nu} + K K_{\mu\nu} - K_\mu^\rho K_{\rho\nu} \\ - \frac{1}{2}\gamma_{\mu\nu}\left(K^2 - K^{\alpha\beta}K_{\alpha\beta}\right) - {}^{(5)}R_{y\mu y\nu}, \end{aligned} \tag{5}$$

where $K_{\mu\nu}$ is the extrinsic curvature tensor of the $y =$ constant hypersurfaces. Taking account of Z_2 symmetry at $y = 0$, the Israel's junction condition gives

$$K_{\mu\nu}(y = +\epsilon) = -4\pi G_5\left(S_{\mu\nu} - \frac{1}{3}\gamma_{\mu\nu} S\right). \tag{6}$$

Substituting the above two equations into Eq. (1), we obtain

$${}^{(4)}G_{\mu\nu} = 8\pi G_4 T_{\mu\nu} + (8\pi G_5)^2 \pi_{\mu\nu} - E_{\mu\nu}, \tag{7}$$

where the 4D effective Newton's constant is given by

$$G_4 = \frac{4\pi G_5^2 \sigma}{3} = \frac{G_5}{\ell}, \tag{8}$$

and $E_{\mu\nu}$ is a projected Weyl tensor defined by

$$E_{\mu\nu} = {}^{(5)}C_{y\mu y\nu}. \tag{9}$$

$\pi_{\mu\nu}$ is a tensor quadratic in $T_{\mu\nu}$ whose explicit form is given in Shiromizu *et al.* (2000). From the 5D conservation law for the localized matter fields, the 4D effective conservation law for the matter fields

$${}^{(4)}D_\nu T_\mu^\nu = 0 \tag{10}$$

follows, which also implies

$${}^{(4)}D_\nu E_\mu^\nu = (8\pi G_5)^2\, {}^{(4)}D_\nu \pi_\mu^\nu, \tag{11}$$

owing to the Bianchi identity.

If we can neglect the last two terms in Eq. (7), the dynamics of 4D general relativity is recovered. We can easily evaluate the order of magnitude of the $\pi_{\mu\nu}$ term, which is quadratic in $T_{\mu\nu}$. This term is smaller by the factor of $T_{\mu\nu}/\sigma$ than the first term $8\pi G_4 T_{\mu\nu}$. Therefore it is guaranteed that this $\pi_{\mu\nu}$ term can be neglected at a law energy. On the other hand, we have not derived any equation which completely determines the evolution of $E_{\mu\nu}$. Equation (11) is not sufficient in general, and the equations that determine $E_{\mu\nu}$ are essentially 5D. Namely, they are not obtained in the form of a 4D effective theory.

4. LINEAR PERTURBATION

As we have reviewed in the previous section, we need to solve a 5D equation in order to fully determine the evolution of the metric induced on the brane. Since solving 5D equation in general is not easy, we consider linear perturbations of the RS model. To discuss metric perturbations in the bulk, the RS gauge is convenient. In this gauge, y-components of metric perturbations are set to be zero: $h_{ya} = 0$, and also $h_{\mu\nu}$ satisfies the transverse and traceless conditions: $h^{\nu}_{\mu,\nu} = h^{\nu}_{\nu} = 0$. The homogeneous equations for bulk metric perturbations become

$$\left[-\partial_z^2 + V(z)\right]\psi_{\mu\nu} = \eta^{\rho\sigma}\partial_\rho\partial_\sigma\psi_{\mu\nu}, \tag{12}$$

where $\psi_{\mu\nu} = \sqrt{|z| + \ell}\, h_{\mu\nu}$ and

$$V(z) = \frac{15}{4(|z| + \ell)^2} - 3\ell^{-1}\delta(z). \tag{13}$$

The solution of Eq. (12) can be found in the form of $\psi_{\mu\nu} \propto u_m(z)\, e^{ik_\mu x^\mu}$. The separation constant $m^2 = -k_\mu k^\mu$ can be understood as the mass of the effective 4D field which corresponds to the mode $u_m(z)$. The equation that $u_m(z)$ satisfies is $[-\partial_z^2 + V(z)]u_m(z) = m^2 u_m(z)$. The solution of this equation with the Z_2-symmetry is $u_m(z) = N_m\sqrt{|z| + \ell}(J_1(m\ell)Y_2(m(|z| + \ell)) - Y_1(m\ell)J_2(m(|z| + \ell)))$. The normalization N_m determined so as to satisfy $2\int_\ell^\infty u_m(z)u_{m'}(z)\, dz = \delta(m - m')$ is given by $N_m = \sqrt{m/2}/\sqrt{J_1(m\ell)^2 + Y_1(m\ell)^2}$.

The basic feature of these wave functions can be understood without resorting to the explicit form of the solution. Note that the 5D metric given in Eq. (4) satisfies 5D Einstein equations even if we replace the Minkowski metric $\eta_{\mu\nu}\, dx^\mu\, dx^\nu$ in it with any vacuum solution of 4D Einstein equations. Corresponding to this way of constructing a solution, there is a discrete mass spectrum at $m = 0$ with the wave function $h_{\mu\nu} \propto 1/z^2$. We call it zero mode. This zero mode wave function is nodeless. Hence, there is no bound state for $m^2 < 0$. On the other hand, the potential $V(z)$ asymptotically goes to zero. Hence, the mass spectrum is continuous for $m^2 > 0$. The potential $V(z)$ has barrier near the brane with the height of $O(\ell^{-2})$. For the modes with $0 < m \lesssim \ell^{-1}$, the wave function is suppressed near the brane because of this potential barrier. For the modes with $m \gtrsim \ell^{-1}$, their excitation is kinematically suppressed. Therefore the zero mode is the only active degrees of freedom, and it is a massless and spin-2 field in the language of the 4D effective theory. Hence, 4D general relativity is expected to be recovered at least at the linear level.

Linear metric perturbations induced on the brane were first correctly evaluated in Garriga and Tanaka (2000). The result is summarized as

$$h_{\mu\nu} = -16\pi G_5 \int d^4x' G(x, x')\left(T_{\mu\nu} - \frac{1}{3}\gamma_{\mu\nu} T\right) + \frac{8\pi G_5 \ell^{-1}}{3} \gamma_{\mu\nu}{}^{(4)}\Box^{-1} T, \tag{14}$$

where

$$G(x, x') = -\int \frac{d^4k}{(2\pi)^4}\, e^{ik_\mu(x^\mu - x'^\mu)} \left[\frac{z^{-2} z'^{-2} \ell^{-1}}{k^2 - (\omega + i\epsilon)^2} + \int_0^\infty dm \frac{u_m(y) u_m(y')}{m^2 + k^2 - (\omega + i\epsilon)^2}\right], \tag{15}$$

is the 5D scalar Green function. If we assume the static and spherically symmetric configuration of the matter source localized on the brane, the gravitational field outside the matter distribution is evaluated as (Garriga and Tanaka, 2000; Giddings *et al.*, 2000)

$$h_{00} \approx \frac{2G_4 M}{r}\left(1 + \frac{2\ell^2}{3r^2}\right), \qquad h_{ij} \approx \frac{2G_4 M}{r}\left(1 + \frac{\ell^2}{3r^2}\right). \tag{16}$$

The correction to 4D general relativity is suppressed by the ratio between the 5D curvature scale ℓ and the distance from the center of the star r. The correction to the gravitational field inside the star also stays small by the factor of $O(\ell^2/r_\star^2)$, where $r_\star$ is the typical size of the star. If we neglect the contribution due to massive modes ($m^2 > 0$) in the Green function (15), Eq. (14) exactly reduces to the results for the linearized 4D general relativity.

5. NONLINEAR PERTURBATION

At the linear level, perturbations of the RS model can be expressed as a 4D effective theory with an infinite tower of massive gravitons.[2] However, we will notice that the asymptotic behavior of the wave function at large z is not very regular. The zero mode wave function behaves as

$$h_{\mu\nu}(\text{zero mode}) \approx \frac{1}{z^2}. \tag{17}$$

[2] We do not consider here an alternative possibility to describe the model as a 4D higher derivative theory (Chamblin *et al.*, 2000).

An invariant obtained by contracting the Weyl tensor with itself behaves as $C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma} \approx z^4$. Thus, the infinity in the direction of the extra dimension is a curvature singularity. For the massive modes the situation is worse. The wave function behaves as

$$h_{\mu\nu}(\text{massive mode}) \approx \frac{1}{\sqrt{z}}. \tag{18}$$

Hence, the same invariant more severely diverges as $C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma} \approx z^7$.

However, such a divergence does not indicate the breakdown of the perturbation analysis. The perturbed metric induced by the matter fields on the brane consists of a superposition of various modes. For the static case, the 5D Green function is approximately evaluated in Garriga and Tanaka (2000). The result clearly showed that perturbations at large z are regular. Also in the dynamical cases, the asymptotic regularity of perturbations was shown in Tanaka (2000). An interesting point which we wish to stress here is that perturbations become regular only after summation over all massless and massive modes.

Non-linear perturbations in this model are more complicated. If we adopt the picture of the 4D effective theory with a tower of massive gravitons, one may think that the higher order perturbations can be treated by taking into account the effective coupling between gravitons with various masses. However, this approach does not seem to work. Let us consider the three-point interaction vertex. The effective coupling constant between various massive gravitons will be obtained by expanding the action ($\propto {}^{(5)}R$) to the third order with respect to the metric perturbation h_{ab} and integrating out the dependence on the extra dimension. The object calculated in this way will have the form

$$\int d^4x \int_{\ell}^{\infty} dz \sqrt{-g} g^{**} g^{**} g^{**} g^{**} h_{**,*} h_{**,*} h_{**}. \tag{19}$$

The asymptotic behavior of respective component is given by $\sqrt{-g} \sim z^{-5}$ and $g^{**} \sim z^2$. If we substitute the massive mode wave function, h_{**} and also $h_{**,*}$ are $\sim z^{-1/2}$. Hence, the integrand of (19) behaves as $\propto z^{3/2}$, and the z-integration does not converge. Therefore we cannot define the effective coupling constant well in this manner.

Nevertheless, this does not directly imply any catastrophe at least at the classical level. For the static and spherically symmetric configurations in the 4D sense, second-order perturbations were calculated, showing that the perturbations behave well, and the correction to 4D general relativity is suppressed by the factor of $O(\ell^2/r_\star^2)$ (Giannakis and Ren, 2001; Kudoh and Tanaka, 2001). The approximate reproduction of the results for 4D general relativity is also confirmed numerically in Wiseman (2001), in which the strong gravity regime was also investigated. Hence, one may be able to conclude that the gravity in the RS infinite brane world is well approximated by 4D general relativity, although non-linear perturbations have not been computed in dynamical cases at all. We would like to stress that the effective coupling constant between massive gravitons is not well defined in this model. Nevertheless, it seems that the model approximately recovers 4D general relativity.

6. BLACK HOLE AND AdS/CFT CORRESPONDENCE

In the preceding section we have observed that the induced metric on the RS brane mimics the results of 4D general relativity well. This seems to work even in the strong gravity regime. However, no black hole solution which is asymptotically AdS has not been found so far.

In Chamblin *et al.* (2000), a black string solution given by

$$ds^2 = \frac{\ell^2}{(|z| + \ell)^2}\left[dz^2 + q^{(4)}_{\mu\nu}\, dx^\mu\, dx^\nu\right] \tag{20}$$

was discussed. Here $q^{(4)}_{\mu\nu}\, dx^\mu\, dx^\nu$ is the usual 4D Schwarzschild metric. The induced geometry on the brane at $z = 0$ is exactly 4D Schwarzschild space–time. However, the asymptotic value of $C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma}$ behaves as $\propto z^4 r^{-6}$, where r is the Schwarzschild radial coordinate. If we take the $z \to \infty$ limit for a fixed r, this curvature invariant diverges[3]. Also, this configuration is unstable (Gregory, 2000; Gregory and Laflamme, 1993; Horowitz and Maeda, 2001). Hence, the black string solution will not be an appropriate candidate for the final state of the gravitational collapse in the brane world. A conjecture raised in Chamblin *et al.* (2000) is that there will be a configuration called "black cigar" for which the event horizon is localized near the brane.

However, as we have mentioned above, no black hole solution which is asymptotically AdS has been found so far, although there were several works aiming at finding it. Here we give rise to a suspect on the existence of the brane black hole based on the argument of AdS/CFT correspondence. For the introduction to AdS/CFT correspondence, we follow Hawking *et al.* (2000) and Shiromizu and Ida (2001). Here we consider the RS model without matter fields localized on the brane. The AdS/CFT correspondence implies the relation

$$S_{\mathrm{RS}} = S^{(4)}_{\mathrm{EH}} + 2W_{\mathrm{CFT}}, \tag{21}$$

where W_{CFT} is the connected Green function with a high frequency cutoff for certain 4D CFT fields evaluated on the background metric induced on the brane, and

$$S^{(4)}_{\mathrm{EH}} = -\frac{\ell}{16\pi G_5}\int d^4x\, \sqrt{-q}^{(4)} R \tag{22}$$

[3] More detailed discussions are given in Chamblin *et al.* (2000)

is the ordinary 4D Einstein–Hilbert action for the induced metric on the brane. This formula indicates that the RS infinite braneworld is equivalently described by 4D general relativity coupled to conformal fields. The number of degrees of freedom of the conformal fields is $O(\ell^2/G_4)$, which is supposed to be large. It is known that the quantum effect by means of 4D CFT corresponds to the classical effect due to the bulk graviton in the 5D picture. This fact can also be understood in the following way. Let us consider the energy–momentum tensor of the 4D CFT due to the vacuum polarization effect induced by the curved geometry. In this case, the contribution to the energy–momentum tensor from each field will be $O(1/L^4)$, where L is the characteristic length scale of the space–time curvature. Thus, the vacuum polarization part of the energy–momentum tensor ${}^{(4)}T^{(Q)}_{\mu\nu}$ in total will become $O(\ell^2/G_4L^4)$, where we have taken into account the number of degrees of freedom. Hence, from 4D Einstein equations ${}^{(4)}\Box h_{\mu\nu} \approx G_4{}^{(4)}T_{\mu\nu}$, the additional metric perturbation caused by this energy–momentum tensor is estimated to be $h^{(Q)}_{\mu\nu} = O(\ell^2/L^2)$. On the other hand, the effective energy–momentum tensor induced by the quantum effect in the 5D point of view is also given by the curvature scale of the space–time. When we discuss in the 5D picture, there are two characteristic length scales, ℓ and L. We denote both of them by $\tilde{L}$ without distinguishing them. Then, we will have ${}^{(5)}T^{(Q)}_{\mu\nu} \approx \tilde{L}^{-5}$. Then, from 5D Einstein equations ${}^{(5)}\Box h_{\mu\nu} \approx G_5{}^{(5)}T_{\mu\nu}$, we will obtain $h^{(Q)}_{\mu\nu} = O(\tilde{L}^{-3}G_5) = O(\tilde{L}^{-2}G_4)$. Note that the number of degrees of freedom is a few in this case. Since the power of G_4 does not coincide in the above two expressions for $h^{(Q)}_{\mu\nu}$, it is almost impossible to expect that the contribution due to the quantum effect in 5D picture corresponds to that in the 4D CFT picture. This mismatch comes from the large number of CFT fields of $O(\ell^2/G_4)$.

Let us apply the argument of the AdS/CFT correspondence to the formation of a black hole in the RS braneworld. In the 4D CFT picture, a black hole is formed in the presence of a large number of conformal fields. Then, the black-reaction due to Hawking radiation will be much more efficient than in the ordinary 4D theory by the factor of ℓ^2/G_4. If the statement given above is also valid in this situation, the quantum back-reaction due to Hawking radiation in 4D picture must be described as a classical dynamics in the 5D picture. When we look at this situation as a 5D process, the black hole evaporates as a classical process. This may imply that there is no stationary black hole solution in the 5D RS model.

However, there may be several arguments against the above statement. One of them is as follows. In the absence of matter fields on the brane, the effective Einstein equations (7) become ${}^{(4)}G_{\mu\nu} = -E_{\mu\nu}$. Hence, $G^{\mu}_{\mu} = 0$ because $E_{\mu\nu}$ is traceless. On the other hand, the trace part of the energy–momentum tensor of CFT is determined by the trace anomaly, and it is evaluated as $T^{\rm CFT} = -(\ell^2/32\pi G_4){}^{(4)}G_{\mu\nu}{}^{(4)}G^{\mu\nu}$. Then the AdS/CFT correspondence will imply that ${}^{(4)}G_{\mu\nu}{}^{(4)}G^{\mu\nu} = 0$ when the energy–momentum tensor of the matter fields on the brane $T_{\mu\nu}$ vanishes. However, we know the asymptotic behavior of metric perturbations far from the matter distribution in the linear perturbation (16). It is known that the metric outside any spherical matter distribution should coincide with Schwarzshild metric when 4D vacuum Einstein equations ${}^{(4)}G_{\mu\nu} = 0$ are imposed. Hence, ${}^{(4)}G_{\mu\nu} \neq 0$ for the metric (16). For this static metric with spherical symmetry, ${}^{(4)}G_{\mu\nu}$ has a diagonal form. Therefore we find ${}^{(4)}G_{\mu\nu}{}^{(4)}G^{\mu\nu} \neq 0$. Nevertheless, the energy–momentum tensor of the matter field is vanishing there. This might be a contradiction. However, when we quote expression (16), we are assuming the presence of matter fields on the brane. If the AdS/CFT correspondence works only in the setup without additional matter fields, the above argument is not a contradiction.

Another evidence against the absense of black hole solution is the existence of a static black hole solution when the brane is 3D. This solution was found by Emparan *et al.* (2000). The 3D metric induced on the brane looks similar to the 4D Schwarzshild black hole:

$$ds^2 = -\left(1 - \frac{2\mu\ell}{r}\right) dt^2 + \left(1 - \frac{2\mu\ell}{r}\right)^{-1} dr^2 + r^2\, d\varphi^2. \tag{23}$$

An important difference from the 4D Schwarzshild black hole is that the period of identification in φ-direction is not 2π but $\Delta\varphi \approx \frac{4\pi}{3(2\mu)^{1/3}}$, where we assumed that $\mu \gg 1$.

For this black hole, we have

$$E_{\mu\nu} = \frac{\mu\ell}{r^3}\, \mathrm{diag}(1, 1, -2). \tag{24}$$

If we apply the AdS/CFT correspondence, the energy–momentum tensor of CFT is estimated to be $T^{\mathrm{CFT}}_{\mu\nu} \approx -(8\pi G_3)^{-1} E_{\mu\nu}$. To maintain a static configuration under the existence of Hawking radiation, there should exist thermal bath which supplies the incoming energy flux to balance with the Hawking radiation. However, $T^{\mathrm{CFT}}_{\mu\nu}$ decays too fast for large r to explain this thermal bath. Nevertheless, this energy–momentum tensor is finite on the event horizon. (In the even D-dimensional cases, the energy–momentum tensor in a static quantum state has a component from the thermal radiation which behaves as $T^{\nu}_{\mu} \approx \theta^D g_{00}^{-D/2}$, while the remaining part decays fast for large r. By setting θ equal to the Hawking temperature, the total energy–momentum tensor becomes regular on the event horizon. If we require fast decay of the energy–momentum tensor for large r, we need to subtract this thermal contribution. Hence, the energy–momentum tensor in a static vacuum with fast decay is considered to diverge on the event horizon. In the present case, the Hawking temperature determined from the regularity of the Euclideanized manifold is $\theta = 1/(8\pi\mu\ell)$.

However, as we have not understood well the vacuum polarization of CFT on this background, in a strict sense we cannot say that this example is a contradiction.

It will be worth pointing out that this effective energy–momentum tensor (24) can be understood as the Casimir energy of CFT. This space–time is very compact in the φ-direction. The period in the φ-direction is $r\,\Delta\varphi \approx r/\mu^{1/3}$, while the curvature scale is $L \approx (\mu\ell/r^3)^{-1/2}$, which is much longer than the period in the φ-direction. Hence, the approximation by a cylinder with a fixed radius $\approx r/\mu^{1/3}$ will be good. For such a cylinder, the energy–momentum tensor for one conformal field is given by $\approx(\mu/r^3)\,\mathrm{diag}(1, 1, -2)$. The number of CFT species of $O(\ell/G_3)$ consistently explains the correspondence relation.

7. SUMMARY

In this paper, we reviewed the studies on the gravity in the RS infinite brane world. All the computations performed so far suggest that this model recovers the 4D general relativity as an effective theory induced on the brane. However, any black hole solution with the regular asymptotic behavior has not been obtained. We discussed the possibility that there is no static black hole solution in this model. Applying the argument of the AdS/CFT correspondence to the situation with a black hole, we obtained a statement which supports this interesting possibility. If the black hole solution in the RS infinite braneworld does not really exist, we may be able to use this fact as a probe of the existence of a warped extra dimension. However, the discussion presented in this paper is not sufficiently rigorous to give a definite conclusion to this conjecture. We would like to give further discussion on this issue in future publication.

ACKNOWLEDGMENTS

The author thanks J. Garriga for useful comments and discussions. Also he thanks IFAE for its hospitality. This work was supported in part by the Monbukagakusho Grant-in-Aid No. 1270154 and by the Yamada Science Foundation.

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